行政院國家科學委員會專題研究計畫成果報告

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計畫名稱: **考量收益時間差異及使用避險工具之退休基金最適資產負債動態模型**

執行期限: **90年8月1日至91年7月31日**

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摘要

本研究應用隨機控制理論，以長期規劃之觀點，尋求各期之最適資產配置及提撥金額，為充分反映退休基金管理時所面臨的不確定因素，以隨機微分方程式描述退休基金所累積資產與應計負債的動態隨機性質，建構連續時間的隨機控制模型，並給定衡量資產負債之評估績效，求出最適的基金提撥與資產配置策略。最終以勞動基準法規範下的企業退休金計劃為實証對象，透過動態模擬與數值方法，連結控制理論與情境模擬，藉以檢視某半導體與電子公司固定給付退休基金之最適策略。

關鍵字：最適提撥；資產配置；評估測度；動態規劃。

Abstract

This paper studies the dynamic funding policy and investment strategy for the defined benefit pension schemes with the most comprehensive dynamic models of the pension plan to date. We formulate the optimal decisions of pension plans as a stochastic control problem and solve the problem through dynamics programming. The objective function of our model takes into account of the stability and the solvency of the pension schemes. The assets in the model include one risk-free asset and two risky assets while the considered liabilities are a three-dimensional process. We first investigate the procedures in deriving the optimal strategies under the specified performance measures incorporating asset-liability matching criterions. Then we apply our method to the pension plan of a semi-conductor and electronic company in Taiwan for illustration.

Keywords: optimal contribution, asset allocation, dynamic programming, performance measure.

一、計畫緣由及目的

Conventional pension plan management is constructed under one-period assumption. The pension plan manager searches for an optimal investment decision for the next period, basing on the plan's current positions, current conditions of financial markets, and expectations about future funding, returns, and risks. Such short-seeing mechanism has two drawbacks. First, the aggregation of single-period optimal decisions across periods might not be optimal for multiple periods as a whole. (see e.g., Brennan et al. (1997) and Samuelson (1989, 1990)) Second, single-period decisions have difficulties in simultaneously dealing with the investment and the funding sides of the pension plan, because the linkage between investments and funding appears only in the multi-period setting. Since most pension fund holders are long-term investors and the financial strength of a pension plan depends on both sides of the balance sheet, pension plan management therefore should be considered within multi-period framework.

The method of stochastic control is

promising for multi-period models, especially with the help of high-speed computers. Methods of optimal control solve long-term financial planning problems through global optimization across periods instead of local optimization within a period. The control theory has been developed in engineering fields since 1930s and the applications to economics emerged in 1950s (see Petit (1990) for more descriptions on the development of the literature). Papers along this line such as Samuelson (1969), Merton (1971, 1990), Brennan and Schwartz (1982), Karatzas et al. (1986), Brennan et al. (1997), Boyle and Yang (1997), Brennan and Schwartz (1998), Sorensen (1999), Xia (2001), Wachter (2002), Brennan and Xia (2002) studied the optimal consumption and investment problems under different settings using the stochastic control theory. Although the applicability and popularity of the stochastic control method was somehow hindered by the embedded complexity before, it has become more popular and powerful recently due to the availability of high-speed computing.

The application of the control theory to pension plan management starts from O'Brien (1986, 1987). He constructed a stochastic model for the pension plan and studied the optimal funding policies to achieve target-funding ratios. Cairns (1995, 1996, 2000) further introduced the asset allocation into the controlled process. He studied the optimal investment strategies as well as optimal funding policies to minimize certain quadratic loss functions. Chang (1999, 2000) applied the above theoretical method to a real world case. For a pension plan covering almost three thousand employees, he constructed a dynamic model considering random survivorship of the employee group as well as stochastic asset returns and numerically solved for optimal funding policies over various time horizons. Applications of the control theory to other actuarial problems can be seen in Runggaldier (1998) and Schäl (1998). Runggaldier reviewed the concepts and solution methods, while Schäl focus on the dynamic programming for piecewise deterministic Markov processes.

We in this paper construct one of the most comprehensive dynamic models of the pension plan to date, numerically solve the stochastic control problem, and provide visual illustration of the solved optimal investment and funding strategies. Compared with Cairns (1995, 1996, 2000), we have much richer liability dynamics and have risk-free as well as risky assets. Compared with Chang (1999, 2000), we consider not only the funding policies but also the asset allocations, and consider more risk factors for invested assets. Furthermore, our performance measure (also called loss function) take into account of both the stability of contributions and the security (the funding ratio) of the pension plan. Our method thus can help in practically evaluating the risks associated with specific plan strategies and assess the trade-off between stable contribution and solvency of the plan along the evaluation time horizon. The features of our models are summarized in the following:

1. The dynamics of the plan's demography can be explicitly incorporated into investment decisions under different evaluation time horizons.
2. The optimal funding and investment strategies of the plan can be formulized with specific risk performance measures through a computerized system.
3. The contribution risk and the solvency risk associated with any funding policies and investment strategies can be concretely assessed given any evaluation horizon.

The paper is organized as follows. Section 2 describes the proposed dynamic optimization scheme, starting from the basic framework and followed by the dynamics of invested assets, loss function, and optimal equations. Section 3 explores its practical usefulness on a real world case and provides numerical illustrations. Short conclusions and discussions for future researches are in Section 4.

二、結果與討論

The method of stochastic control is a helpful tool to derive optimal strategies for a finance or insurance portfolio. It represents a significant advancement over one-period models because it can explicitly consider the inter-period dynamics and aim at long-term rather than short-term optimality. Furthermore, dynamic control models can simultaneously handle funding policies and investment decisions of the pension plan. This is also an important advantage because the solvency of the pension plan hinges on the relative sizes of the assets and liabilities that in turn depend on funding and asset returns.

In this study, we establish a dynamic control model for the pension plan and apply it to a real case. Our model is the most comprehensive one so far, by gathering merits from different papers. In establishing the objective function, we refer to Haberman and Sung (1994) and consider the contribution risk (the stability of contributions) and the solvency risk (the security of funds). We enlarge the set of invest able assets to the one containing both risk-free and risky assets, following the work of Brennan et al. (1997). For liabilities, we employ the stochastic simulations in Chang (1999, 2002) that explicitly characterize the participating population of a pension plan. With these dynamics structure and objective function, we derive a system of differential equations and the solutions of these equations represent optimal funding policies and asset allocations. We then apply the theoretical model to a real case for illustration and numerically obtain the optimal solutions. The proposed model can help the pension plan manager better formulate the financing, investment, and risk management decisions.

Future research can improve upon this paper in several ways. First, researchers may want to add short sale constraints into the model since our solved optimal strategies usually involve certain amount of short sales. Most pension funds, however, are not allowed to engage in short sales because the associated downside risk is tremendous. Second, future research may want to consider transaction costs. Transaction costs are going to change the relative advantage of active trading to passive trading and thus might result in different optimal trading strategies. Finally, optimal hedging policies might also be incorporated into our model in the future.

三、計畫成果自評

本文利用決策經濟模型研究最適策略，採用連續時間模型描述基金動態變化，針對利用所推導之最適化理論實証後所得到的結果。

運用控制模型於退休金財務規劃，可突破傳統精算評價方法的限制，它不僅可考慮決策者的主觀目標，而且與風險評估結合，於此退休基金管理的控制架構，風險衡量在決策過程中扮有重要的角色，不論在建構最適資產組合，或者針對策略的效率而言，風險衡量在退休基金管理上提供彈性的評估指標。

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